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Brief communication

Hydrodynamic force on interactive spherical particles due to the wake effect

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1. Introduction

Extensive experimental and theoretical studies are currently under development pursuing to analyze the interaction of two equally sized spherical bodies, whose line of centers is aligned along the flow direction for different values of the Reynolds number, *Re* (Ruzicka, 2000). In a recent paper Zhang and Fan (2002, herein referred to as ZF2002) obtained an expression for the hydrodynamic force on an interactive sphere placed in the wake of the leading sphere. They consider the dominant viscous drag force as the only meaningful force, neglecting inertial effects.

For a vertically aligned pair of equally sized spherical bubbles rising at $Re \geqslant 50$ in clean water, the flow structure for the inviscid flow and the viscous flow is very alike (in particular, there is no boundary layer separation). Thus, the inertial effects could be taken into account using the expression for the inviscid repulsive force given by Lamb (1932, Article 138) and Harper (1970, Eq. 2.3), as done by Yuan and Prosperetti (1994, Eq. 5.1) and by Ruzicka (2000, Eq. 8) and Ruzicka (2005, Eq. 2.10). Nevertheless, the boundary conditions at the surface of gas bubbles and solid spheres are different: zero-tangential-stress for the bubble and no-slip condition for the solid sphere. This difference influences the vorticity production at the solid surface and thereby the wake structure. Consequently, it is not possible to extend the potential flow approximation to the pairwise solid sphere interaction.

For an isolated small sphere moving in a non-uniform, unsteady laminar flow, Maxey and Riley (1983) demonstrated that, irrespectively of the *Re* values, the body experiences an inertial hydrodynamic force due to the Lagrangian acceleration of the undisturbed flow. Furthermore, the body decelerates the motion of a part of the surrounding fluid, resulting in an additional effect on the body, that is wholly inertial, the well-known added-mass force (Magnaudet and Eames, 2000; Mei and Klausner, 1992). Therefore, the added mass force and the fluid convective acceleration should be taken into account, even for a fixed pair of spheres in an

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otherwise uniform and steady state flow. Katz and Meneveau (1996), obtained the velocity of a bubble train rising in-line at Re ranging from 0.2 to 35, considering both the Oseen flow and the potential flow as asymptotic limits for the pressure force estimation. However, the flow structure induced by both, the dynamic pressure gradient and the viscous transport, is equivalent in their approach, to consider the fluid inertial force in an undisturbed flow (see Maxey and Riley, 1983, Eq. 21). Zhang and Fan (2003) consider that additionally to buoyancy and drag forces on a trailing bubble in a quiescent liquid, inertia, pressure gradient, added mass and Basset forces should also be taken into account. Nevertheless, they argue that the pressure gradient is negligible in the far wake region, and that the fluid inertia is zero, in order to neglect these contributions. Recently, Ramírez-Muñoz and Soria (2007) studied, from an integral momentum balance as suggested by Landau and Lifshitz (1958), the whole wake effect on a body at intermediate Re. They showed the existence of two contributions to the hydrodynamic force on the trailing sphere: the viscous drag force and the pressure repulsive force due to the pressure liquid decay with the separation distance between the spheres.

Our objective in this Brief communication is to develop an expression for the hydrodynamic force on a fixed trailing solid sphere, considering both inertial and viscous drag forces. This expression fitted experimental data from Zhu et al. (1994), Chen and Lu (1999) and Chen and Wu (2000), at Re values of 52–145, with a relative average error, Er between $3.5\% \leqslant Er \leqslant 7.7\%$ in contrast to the model of ZF2002 where Er is between $8.2\% \leqslant Er \leqslant 27\%$.

2. Two interactive spheres aligned in the viscous flow direction

Let us begin by referring to the experimental development by Zhu et al. (1994), through a top-loading electronic balance. It should be stressed that Chen and Lu (1999) and Chen and Wu (2000) used a similar experimental assembly. Each sphere was held fixed by an attached rod. The total force on the trailing sphere was transmitted to the balance through the supporting rod. The gravitational and buoyancy contributions were subtracted from the total force, yielding the hydrodynamic force that is identified by Zhu et al. as "the total drag force". Their experimental development can be generalized as two equally sized spheres moving through an unbounded Newtonian fluid directed along the spheres line of centers. Let us assume their motion be parallel to the vertical z-axis in a laboratory reference frame, as shown in Fig. 1. In order to simplify the analysis, the following assumptions are considered: (1) the only interaction that is considered between the spheres is

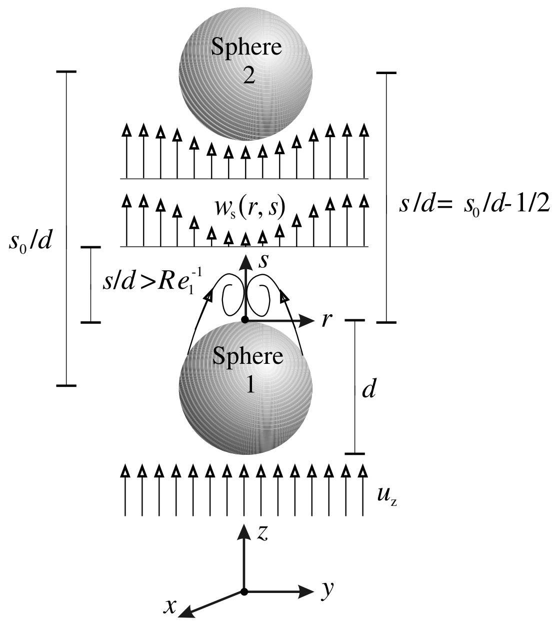

Fig. 1. Hydrodynamic interaction of two spheres arranged in-line.

through the steady non-uniform flow induced by the leading sphere; (2) the Re number is between 24, and 130, after the formation of a standing ring-eddy and before the ring-eddy begins to oscillate (Batchelor, 1967). In this range, the wake flow is laminar, axisymmetric, and steady; (3) the upstream trailing sphere is placed outside of the region of closed streamlines behind the leading sphere. This wake-influenced region extends further as Re increases (Batchelor, 1967; Zhu et al., 1994).

The flow velocity in the leading sphere wake is well approximated for distances from the body such that $s/d > Re^{-1}$ (Batchelor, 1967) by the well-known wake profile

$$\frac{w_z - U_{b1}}{u_z - U_{b1}} = \frac{w_s}{u_s} = 1 - \frac{C_{d1}}{2}\frac{Re_1}{16}\frac{1}{s/d}\exp\left(-\frac{Re_1}{4}\frac{r^2}{sd}\right). \tag{1}$$

Here s and r denote the axial and the radial cylindrical coordinates, respectively, in a reference frame attached to the rear surface of the leading sphere, as shown in Fig. 1. In general, a laboratory reference frame (x, y, z) may be related to the previous one by a Galilean transformation, $z = U_{b1}t + s$, where U_{b1} is the velocity of the leading sphere and t is a time parameter. If the leading sphere is fixed, as in Zhu et al.'s (1994) experiments, this velocity vanishes. In this study, we should consider the general expression taking U_{b1} as an arbitrary constant. $Re_1 = u_s d/v$ and C_{d1} in Eq. (1) are the Reynolds number and the steady drag coefficient applied to the frontal area, respectively. Also, v is the kinematic viscosity and d is the diameter of the sphere. Furthermore, u_z is the uniform flow velocity in the laboratory reference frame, while u_s is the same velocity in the leading sphere reference frame, i.e.

$$u_s = u_z - U_{b1}. \tag{2}$$

The same notation is adopted for the non-uniform flow velocities w_z and w_s.

3. Hydrodynamic force on the interactive sphere

It should be stressed that the total hydrodynamic force is not only the drag force. Even though the upward liquid flow on the leading sphere is assumed as a uniform steady state, the leading sphere wake induces a non-uniform flow on the trailing one. Therefore, we take into account the added mass force and the fluid convective acceleration.

The drag force estimation is performed by using the conventional isolated sphere C_d's correlations, but introducing a proper definition of the reference fluid velocity, as pointed out by Yuan and Prosperetti (1994). ZF2002 defined this reference fluid velocity by considering the radial average wake velocity over the projected area of the trailing sphere. Following the same consideration for the trailing sphere (sub-index 2):

$$F_{d2} = C_{d2}\frac{\pi}{4}d^2\frac{1}{2}\rho(\bar{w}_z - U_{b2})^2, \tag{3}$$

meanwhile for the leading sphere (sub-index 1) we have:

$$F_{d1} = C_{d1}\frac{\pi}{4}d^2\frac{1}{2}\rho(u_z - U_{b1})^2. \tag{4}$$

Here $C_{d1} = 24/Re_1(1 + 0.15Re_1^{0.687})$ (Schiller and Nauman, 1933) valid for $Re < 800$, and a similar expression is used for C_{d2}. In addition, the projected area average of the non-uniform flow velocity is defined as

$$\bar{w}_z = \frac{4}{\pi d^2}\int_0^{d/2} w_z(r, s)2\pi r\,\mathrm{d}r. \tag{5}$$

The local undisturbed average wake velocity, evaluated at the center of the trailing sphere, is obtained from (1) and (5) in dimensionless form as

$$W \doteq \frac{\bar{w}_s}{u_s} = 1 - \frac{C_{d1}}{2}\left[1 - \exp\left(-\frac{Re_1}{16}\frac{1}{s/d}\right)\right]. \tag{6}$$

Moreover, the forces acting on the trailing sphere are parallel to the upward oriented z-axis, so that the body would experience no couples. Therefore, the dimensionless vertical component of the hydrodynamic force (F_{HD}) can be expressed as

$$\frac{F_{\mathrm{HD}}}{F_{\mathrm{d1}}} = \frac{F_{\mathrm{HDI}}}{F_{\mathrm{d1}}} + \frac{F_{\mathrm{d2}}}{F_{\mathrm{d1}}}. \tag{7}$$

Since 1928, Taylor derived the inertial hydrodynamic force (F_{HDI}) on a fixed body immersed in a steady (non-uniform) irrotational flow, when the size of the body is smaller than the length scale of the variations in the undisturbed flow (Auton et al., 1988; Magnaudet and Eames, 2000). Furthermore, if the relative direction of the flow is parallel to one of the axes of permanent motion, this force is further reduced to an inertial contribution, $m_{\mathrm{f}}\bar{w}_z \mathrm{d}\bar{w}_z/\mathrm{d}s$ plus an added mass force $m_{\mathrm{f}}C_M\bar{w}_z \mathrm{d}\bar{w}_z/\mathrm{d}s$, so that

$$F_{\mathrm{HDI}} = m_{\mathrm{f}}(1 + \mathrm{C}_M)\bar{w}_z \frac{\mathrm{d}\bar{w}_z}{\mathrm{d}s}. \tag{8}$$

Here $m_{\mathrm{f}} = \rho\pi d^3/6$, denotes the mass of fluid displaced by the sphere and ρ is the fluid density. C_M is the added mass coefficient and its value for a spherical body is $C_M = 1/2$. Besides, $\bar{w}_z$ and $\mathrm{d}\bar{w}_z/\mathrm{d}s$ are evaluated at the body center for the undisturbed flow and $\bar{w}_z\mathrm{d}\bar{w}_z/\mathrm{d}s$ is the z-axis convective acceleration due to non-uniformity. Therefore

$$\frac{F_{\mathrm{HDI}}}{F_{\mathrm{d1}}} = (1 + \mathrm{C}_M)\frac{Re_1}{24}\frac{1}{(s/d)^2} W \exp\left(-\frac{Re_1}{16}\frac{1}{s/d}\right). \tag{9}$$

Substituting (3), (4) and (9) in (7) and considering $C_M = 1/2$, we get the following expression for the hydrodynamic force ratio on the trailing sphere

$$\frac{F_{\mathrm{HD}}}{F_{\mathrm{d1}}} = W\left[1 + \frac{Re_1}{16}\frac{1}{(s/d)^2}\exp\left(-\frac{Re_1}{16}\frac{1}{s/d}\right)\right]. \tag{10}$$

For the limit $s/d \to \infty$ the hydrodynamic force on the trailing sphere reduces to the drag force of an isolated sphere.

4. Results and discussion

To evaluate the prediction of the expression proposed above, some numerical evaluations of the hydrodynamic force were conducted at the same conditions of the work performed by experimental studies by Chen and Lu (1999) for Re_1 of 52, Chen and Wu (2000) for Re_1 of 54 and Zhu et al. (1994) for Re_1 of 54, 106 and 145. In Fig. 2 we plot $F_{\mathrm{i}}/F_{\mathrm{d1}}$, where F_{i} denotes F_{d2} (from ZF2002) or F_{HD} (from Eq. (10)) as a function of the center-to-center distance between both spheres, s_0/d (Fig. 1), with $s_0/d = s/d + 1/2$ for a fixed Re_1.

The better fitting of the present model to the experimental data than the one by ZF2002 is apparent in Fig. 2. It should be pointed out that to perform their computations, ZF2002 define a separation distance between the rear surface of the leading sphere and the frontal surface of the trailing one. Nevertheless, we found better to compute the forces on the sphere considering the separation distance up to the place coincident with the trailing body center. This consideration is frequently used in the literature (cfr. Maxey and Riley, 1983; Auton et al., 1988; Magnaudet and Eames, 2000). In order to estimate the statistical average error of both models using the experimental data, the relative average error is introduced as (Bevington and Robinson, 1992),

$$\%Er = \sum_i^N \frac{1}{N}\frac{\left[(\mathrm{Exp})_i - (\mathrm{Theo})_i\right]}{(\mathrm{Exp})_i} \times 100. \tag{11}$$

The results obtained with our model, given by Eq. (10), show that the agreement between the experimental and theoretical values of the force ratio is within an error between 3.5% for $Re_1 = 106$ and 7.7% for $Re_1 = 52$; meanwhile, the ZF2002's model gives an error between 8.2% ($Re_1 = 106$) and 27% ($Re_1 = 54$).

The origin of this difference is due to two contributions: (1) the inertial forces inclusion, whose relative importance increases as the separation distance is reduced, up to a 13.7% of the total hydrodynamic force

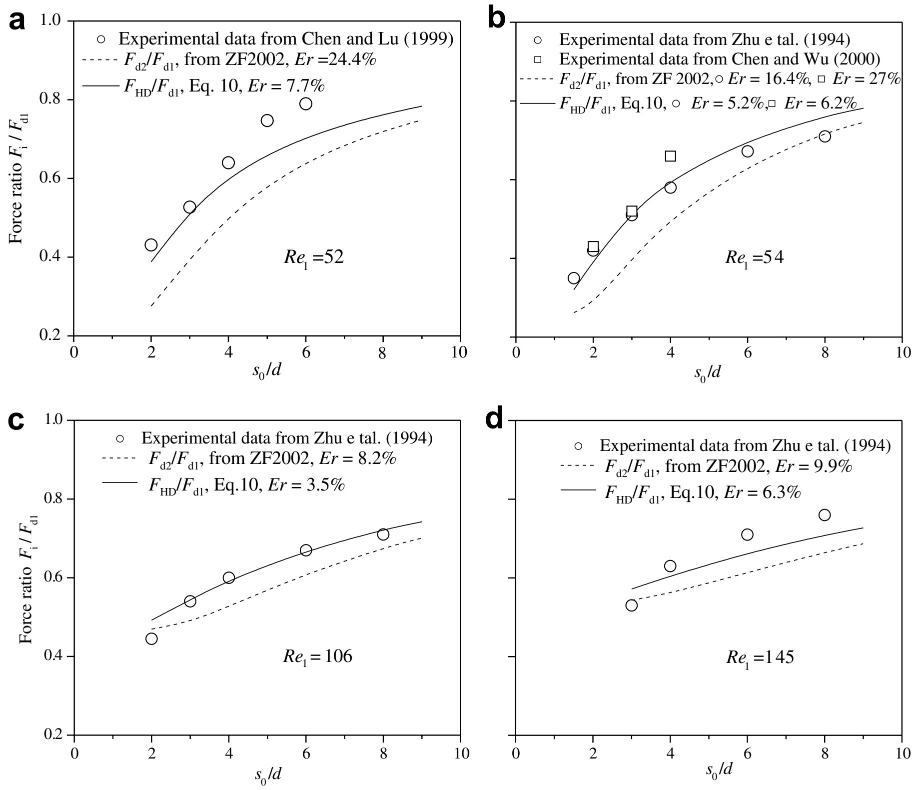

Fig. 2. Force ratio as a function of the dimensionless distance between the sphere centers for (a) $Re_1 = 52$, (b) $Re_1 = 54$, (c) $Re_1 = 106$ and (d) $Re_1 = 145$.

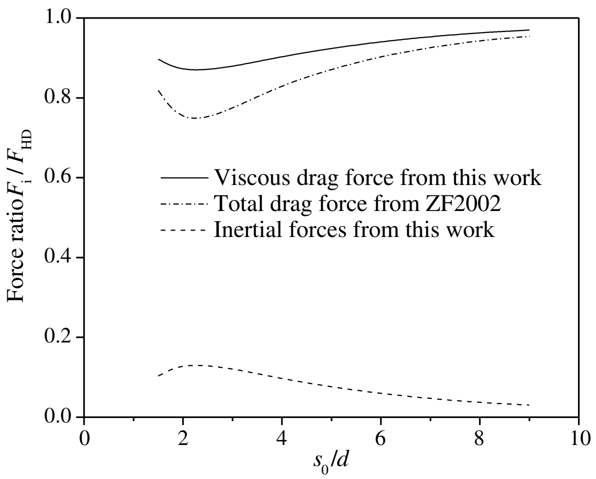

Fig. 3. Contributions to the hydrodynamic force on the trailing body from present work and ZF2002, at $Re_1 = 54$.

at $Re = 54$ and $s/d = 2$; as can be observed in Fig. 3. The computations of both forces were performed here with our model. (2) As referred above, the ZF2002's model performed computations for separation distances defined from the rear surface of the leading sphere to the front surface of the trailing one. Consequently, their computations result in an additional drag force underestimation. Their predictions at $Re = 54$ and $s/d = 2$

yield a total discrepancy up to 27% of the total hydrodynamic force, as can be seen in Fig. 3 and inferred from Fig. 2b.

Thus, the model of ZF2002 underestimates the total wake effect on the trailing sphere, not only because of their separation distance choice, but also because of their neglecting inertial effects. In our model, much of this discrepancy is removed by the assumption that the leading body wake effect on the trailing body involves not only a quasi-steady drag reduction, but also the inertial forces (i.e. the fluid convective acceleration and the added mass force). When these forces are accounted for, even by using the laminar wake similarity solution, a significant improvement is obtained on the model predictions.

5. Conclusions

In this work, we revisited the estimation of hydrodynamic forces on a trailing sphere in a two-body system. The hydrodynamic model used in this work considered both, the inertial forces and the dominant viscous drag. Steady state wakes do not depend explicitly on time, i.e. their local acceleration is zero. Nevertheless, the convective acceleration is closely related to the spatial non-uniformity and should be considered through the inertial effects in the hydrodynamic force on the trailing sphere.

The results obtained with this model show that the agreement between the experimental and theoretical values for the hydrodynamic force is within a relative error between 3.5% and 7.7%. We obtained a better prediction than the models neglecting inertial forces. This fact reinforces the point of view advocated above, meaning that the wake acceleration and the added-mass are important forces in the hydrodynamic interaction.

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